# Rumi Numeral System Symbols,

Additional characters proposed to Unicode

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## 1 Introduction

A special numeral system  $rumi^1$  has been in use in North Africa since the X<sup>e</sup> century. It remained in use until the XVII<sup>e</sup> century. This system has been especially used in the administration of the city of Fez in Morocco. It has also been used in Al-Andalusians, Spain, starting from the XII<sup>e</sup> century. The forms of the digits are quiet different from the Arabic<sup>2</sup> or the Arabic-Indic<sup>2</sup> digits in use today. The system of numeration was decimal, but not really positional. rumi use some special symbols (see Table 1, Table 2, Table 3 and Table 4). Some examples are available (see Table 5, Table 6 and Table 7). This system is also known as "zimam letters, Roman, Fez letters" (namely Hrwf al-zmAm, al-rumi, Hrwf fAs or also rasm al-zmam, Qalam al-rumi, Qalam al-fAsy).

### 2 Description

rumi numeral system has been described by many researchers and there is many studies about it. A deailed bibliography is presented in Figure 1 (see some figures from this bibliography in the end). We have adopted the one

<sup>&</sup>lt;sup>1</sup>using Transtec Transliteration

http://www.ucam.ac.ma/fssm/rydarab/doc/communic/transtec.pdf

<sup>&</sup>lt;sup>2</sup>the identifier name used by The Unicode Consortium http://www.unicode.org

described by the mathematician Ibn Al-Banna (1256-1321, Marrakech) in his famous book [1] "Abstract of using rumi in calculus" in Arabic (namely Al-YqtDAb mn al-Eml b-al-rwmI fI al-HsAb).

rumi use some special symbols for digits:

- rumi ones are(see Table 1):  $\mathbf{J}, \mathbf{J}, \mathbf{K}, \mathbf{K}, \mathbf{\Lambda}, \mathbf{T}, \mathbf{\Gamma}, \mathbf{T}, \mathbf{J}, \mathbf{J}, \mathbf{S}$ .

Multiples of thousand are represented by adding a slash under the based number:

- rumi thousands are noted by one bar under the number (see Table 5) (ex.,  $\checkmark$  for three thousands);
- rumi million are noted by two bars under the number (see Table 6) (ex.,  $\checkmark$  for three million);
- and so on.

Fraction is represented by adding a slash symbol separating the numerator from the denominator:

- rumi fractions are generally noted by (see Table 7):
- Some special fractions are also noted by (see Table 4): 3, //, /, /, //,

### 3 proposition

These symbols are proposed to be included in Unicode Standard. There names are descriped in rumi ones digits (see Table 8), rumi tens digits (see Table 9) rumi handreds digits (see Table 10) and rumi specila fractions (see Table 11).

As many manuscripts in studying use these symbols, we need to encode them. We are working on studying and translating to english the Ibn Al-Banna manuscript Al-YqtDAb mn al-Eml b-al-rwmI fI al-HsAb. A computer system for transforming numbers from and to rumi numeral system is also in development.

The **rumi** and **AntiSym** fonts available, includes all these characters. In **rumi** font, used here, the shapes of the reference glyphs are scanned from [1]. It's in OpenType format [3] and converted in METAFONT as a LATEX package [4]. In **AntiSym** font, glyphs are drawing by hand in METAFONT as a LATEX package [2].

The shapes of the reference glyphs used are not frozen. They are continually being improved in *Multilingual scientific e-document processing* Project at Al-khawarizmi Atelier.

Some boxes are add to some symbols in Figures in order to emphases them and understand the purpose of the samples.

More information about this presentation is available in [5].

# References

- [1] Ibn Al-Banna (1256-1321, Marrakech), Abstract of using rumi in calculus, in Arabic.
- [2] Arabic mathematical old symbols package antisym for LATEX, http://www.ucam.ac.ma/fssm/rydarab/system/zip/antisym.zip.
- [3] rumi numeral system font in OpenType, http://www.ucam.ac.ma/fssm/rydarab/doc/unicode/rumi.ttf.
- [4] rumi numeral system as a package for LATEX, http://www.ucam.ac.ma/fssm/rydarab/system/zip/rumi.zip.
- [5] Azzeddine Lazrek, Arabic mathematical symbols for Unicode, http://www.ucam.ac.ma/fssm/rydarab/english/unicode.htm.



Table 1: rumi ones symbols



Table 2: rumi tens symbols

100	200	300	400	500	600	700	800	900
e	ケ	હ	¥	9	ð	4	7	Ľ

Table 3: rumi handreds symbols

1/2	1/4	1/3	2/3
ŕ	7	مر	*

Table 4: rumi special fractions symbols

1000	2000	3000	4000	5000	6000	7000	8000	9000
2	5	~~	^	7	<	2	5	ىك
10000	20000	30000	40000	50000	60000	70000	80000	90000
٤	w	٦	بح	SV	2	0	٢	2
100000	200000	300000	400000	500000	600000	700000	800000	900000
e	ケ	٤	メ	2	*	4	7	2

Table 5: rumi thousands examples

4

1000000	2000000	3000000	4000000	5000000	6000000	7000000	8000000	9000000
2	5	~~	<u>^</u>	ケ	<u>~</u>	2	5	25
10000000	2000000	30000000	40000000	50000000	60000000	70000000	8000000	9000000
<b>د</b>	w	لم	<u>٣</u>	<u>sv</u>	٤	0	ل	Z
10000000	200000000	300000000	40000000	500000000	600000000	700000000	80000000	900000000
<b>_</b>	5	٤	<b>×</b>	2	*	4	7	<u>Ľ</u>

Table 6: rumi millions examples



Table 7: rumi fractions examples

J	RUMI DIGIT ONE
	$\approx$ 0031 1 digit one
	$\approx 0661$
5	RUMI DIGIT TWO
_	$\approx$ 0032 2 digit two
	$\approx$ 0662 $\Upsilon$ Arabic-Indic digit two
3	RUMI DIGIT THREE
• -	$\approx$ 0033 3 digit three
	$\approx$ 0663 <sup>°</sup> Arabic-Indic digit three
^	RUMI DIGIT four
	$\approx$ 0034 4 digit four
	$\approx 0664$ $\pounds$ Arabic-Indic digit four
4	BUMI DIGIT FIVE
(	$\sim 0035.5 \text{ digit five}$
	$\approx 0665$ • Arabic-Indic digit five
/	BUMI DIGIT SIX
•	$\approx 0036.6 \text{ digit six}$
	$\approx 0666$ \ Arabic-Indic digit six
2	RUMI DIGIT SEVEN
~	$\approx 0037$ 7 digit seven
	$\approx 0667 \vee \text{Arabic-Indic digit seven}$
4	DIMI DICIT FICUT
	RUMI DIGIT EIGHT
	$\approx 0038$ 8 digit eight $\sim 0668$ A rabie India digit eight
~	
2	RUMI DIGIT NINE
	$\approx 0039$ 9 digit nine
	$\approx$ 0669 9 Arabic-Indic digit nine
	Table 8: RUMI ones digits

۲	RUMI DIGIT TEN
	$\bullet$ used as a symbol with a numeric value of 10
W	RUMI DIGIT TWENTY
	$\bullet$ used as a symbol with a numeric value of 20
2	RUMI DIGIT THIRTY
• •	$\bullet$ used as a symbol with a numeric value of 30
٣	RUMI DIGIT FORTY
	$\bullet$ used as a symbol with a numeric value of 40
ſ	RUMI DIGIT FIFTY
-	$\bullet$ used as a symbol with a numeric value of 50
Ł	RUMI DIGIT SIXTY
	$\bullet$ used as a symbol with a numeric value of 60
0	RUMI DIGIT SEVENTY
	$\bullet$ used as a symbol with a numeric value of 70
6	RUMI DIGIT EIGHTY
_	$\bullet$ used as a symbol with a numeric value of 80
4	DUMI DICIT NINETY
6	• used as a symbol with a numeric value of 00
Ø	• used as a symbol with a numeric value of 90 RUMI DIGIT HUNDRED
5	• used as a symbol with a numeric value of 100
	• used as a symbol with a numeric value of 100

Table 9: RUMI tens digits

RUMI DIGIT TWO HUNDRED
• used as a symbol with a numeric value of 200
RUMI DIGIT TREE HUNDRED
• used as a symbol with a numeric value of 300
RUMI DIGIT FOUR HUNDRED
• used as a symbol with a numeric value of 400
DUMI DICIT FINE HUNDED
RUMI DIGIT FIVE HUNDRED
• used as a symbol with a numeric value of $500$
RUMI DIGIT SIX HUNDRED
• used as a symbol with a numeric value of 600
RUMI DIGIT SEVEN HUNDRED
• used as a symbol with a numeric value of 700
RUMI DIGIT EIGHT HUNDRED
• used as a symbol with a numeric value of 800
RUMI DIGIT NINE HUNDRED
$\bullet$ used as a symbol with a numeric value of 900

Table 10: RUMI handreds digits



/r

بهم/

#### RUMI FRACTION ONE HALF

- $\bullet$  used as an other symbol with a numeric value of 1/2
- $\approx$  00BD 1/2 vulgar fraction one half

### RUMI FRACTION ONE QUARTER

- $\bullet$  used as a symbol with a numeric value of 1/4
- $\approx$  00BC 1/4 vulgar fraction one quarter

### RUMI FRACTION ONE THIRD

- $\bullet$  used as a symbol with a numeric value of 1/3
- $\approx$  2153 1/3 vulgar fraction one third



#### RUMI FRACTION TWO THIRDS

- $\bullet$  used as an other symbol with a numeric value of 2/3
- $\approx$  2154 2/3 vulgar fraction two thirds

Table 11: RUMI special fractions

[2] ابن البا المراكضي، اقتضاب في العمل بالرومي، مخطوط بالخزانة الوطنية بالرباط تحت رقم ف 416 محموعة ص425 -432 [3] اهمد ابن العياشي سكوج، إرغاد المتعلم والناسي في صفة أشكال القلم الثاسي، طبعة فاس، 1316هـ (المكتبة الوطنية 17000989). [4] قاسم بن أهد السامرائي، الأرقام في المشرق عربية النجار وفي النرب الأورى سنسكريتية هندية الدئار، عالم الكتب، المحلد التاسع عشر، العدد الخامس والسادس مزدوجين عن الرقم العربي، 1998. [10] S. Bartina, Cifras coptas y cultura árabe, Studia Papyrologica 7 (1968), 199-210. [11] Georges S. Colin, De l'origine grecque des "chiffres de Fès" et de nos "chiffres arabes", Journal Asiatique 222 (1933), 193-215. http://laporti.club.fr/maghreb/Journalasiatique.doc http://phrontistery.info/nnsbib.html [12] Rosa Comes, Arabic, Rûmî, Coptic, or merely Greek Alphanumerical Notation? The case of a Mozarabic 10th Century Andalusî Manuscript. Subayl 3 (2002-2003), 157-185. http://chama.fltr.ucl.ac.be/chama2/newsletter 3.htm http://www.ub.es/arab/llibrevs/index3.htm [13] A. González Palencia, Los mozárabes de Toledo en los siglos XII y XIII. 4 vols., Madrid, 1926-1930. http://www.filosofia.org/ave/001/a042.htm http://www.architoledo.org/catedral/archivos/textoarchivo.htm [14] Youcef Guergour, Les différents systèmes de numérotation au Maghreb à l'époque ottomane: l'example des chiffres rûmî. E. Ihsanoglu, A. Diebbar et F. Günergun, eds. Burlington, Science, Technology and Industry in the Ottoman World. Proceedings of the XXth International Congress of History of Science, Liège, 20-26 July 1997, Vol. VI. Brepols. Turnhout, 2000, 67-74. http://www.math.buffalo.edu/mad/AMU/amu\_chma\_23a.html http://phrontistery.info/nnsbib.html www.math.buffalo.edu/mad/AMU/amuchmapdf/amuchma23.pdf [15] Ana Labarta y Carmen Barceló, Números y cifras en los documentos arábigo-hispanos. Córdoba, 1988. http://www.islamval-andalus.org/control/noticia.php?id=477 http://museodeamerica.mcu.es/pdf/anales13/capitulo14.pdf http://www.porticolibrerias.es/c/PS627ARA.pdf http://www.porticolibrerias.es/dept\_estudios\_arabes\_valencia.htm [16] G. Levi della Vida, Appunti e quesiti di storia letteraria araba: Numerali greci in documenti arabo-spagnoli, Rivista degli studi orientali 14 (1933), 249-283. http://web.uni-frankfurt.de/fb13/igaiw/publication/mathematic.html http://cura.free.fr/quing/04bezza.html http://web.uni-bamberg.de/unibib/melcom/Sagaria%20Rossi-1.html [17] J.A. Sánchez Pérez, Sobre las cifras rûmies, Al-Andalus 3 (1935), 97-125. http://www.islamval-andalus.org/control/noticia.php?id=477 [18] Abel REY, A propos de l'origine grecque des "chiffres de Fès" et de nos "chiffres arabes", Revue des Etudes Grecques. Tome 48, 1935, pp. 525-538. [19] Driss Lamrabet, Aperçu sur les systèmes de numération en usage au Maghreb du XVIIe siècle, Publications de la Faculté des Lettres et des Sciences Humaines, Rabat ; Si Colloques et Séminaires nº 104, 2003, pp. 23-37. [20] Adam Gacek, The Arabic Manuscript Tradition: A Glossary of Technical Terms and Bibliography, Brill Academic Pub Published, 2001 https://bookweb.kinokuniya.co.jp/guest/cgi-bin/booksea.cgi?ISBN=9004120610

Figure 1: rumi numeral system in [1] page 1

ille a X 7 425 هرا العراليب العد 24 i Ċ 15 8 ζw C とう Ø ۶ Ŵ 93 4 J)

Figure 2: rumi numeral system in [1] page 1

بسمالأه الرحمن الرحيم صلى الأمعلى سيدنا محمد وعلى أله وصحبه وسلم تسليما

[لاقضاب من العمل بالرومي في الحساب، تأليف الشيخ الفقيه الجليل الفاضل المشارك الأكمل المرحوم أي العاس أخمد بن عمد بن عثمان الأرذي، حرف بابن البناء العددي المراكشي، قدس الله روحه ورخمه بمه وفضله وطوله".

الحمد لله همدا لا انقطاع <sup>ع</sup> لعده <sup>\*</sup> ولا فحاية لحده والصلاة على محمد نبيه وعبده وعلى أله وسلم وضرفه كثيرا <sup>\*</sup>. وبعد، فهذا اقضاب من العمل بالرومي في الحساب على نحو ما وقع <sup>ت</sup>احتيار الرؤساء <sup>ف</sup> من العمال، ومن الله اسأل حسن التوفيق. باب في أ<sup>م</sup>ماء مراتب الأعداد وصفة الرضم بالرومي. اعلم أن المرتبة الأول هي الأحاد النسعة والثانية هي العشرات النسع والثالثة هي المؤن النسع والرابعة هي الألاف النسعة والخامسة هي عشرات

الألاف النسعة والسامسة هي ملثوا الألاف النسع وللسابعة هي آلاف الألاف النسعة وهكذا إلى غير فحاية. ولكل عدد من أعداد المراتب الثلاث الأول رشم يخصه أولها واحد وآخرها تسع مائة ورشوم الألاف وعشرائها ومائيها كرشوم الأحاد وعشرائها ومائيها والفرق بينهما بالتكرار وشكل التكرار خفظة تحت العدد وهذه صورة ذلك كله:

وكذلك آلاف الألاف وعشراقًا ومؤها ترجع إلى رغوم التي قبلها والفرق بينهما التكرار. لكل نوع ما يجب له منه " كذلك إلى غير أفاية. وأما رضوم الكسور فرغم العدد الذي يشتق منه الكسر<sup>11</sup> ويخط فوقه خط يسمى كرسيا ويكتب فوقه عدة الأجزاء التي تشتق منه وكذلك كسور الكسور وهي أمثلة منها رغم نصف **ك**و وثلثين **عُمَ** وثلاثة أرباع محمم وأربعة أخمل محم وغسة



أمدنس على وسنة أسباع مسر وسبعة أكمان سلم وتمانية أنساع <sup>22 علمي</sup> وتسعة أعشار على وقد اصطلح أصحاب العمل على أن يرتموا النصف هكذا من والربع هكذا مسرر والثلث هكذا مسلم والثلثين <sup>مد</sup> هكذا محكم ولا يستعملون الأجزاء التي مقاماتها أكثر من عشرة. وان وقع لهم شيء منها في العمل صرفوه على ما ستعرفه في باب الجمع بعد هذا. والكسور المستعملة عندهم نوعان مضافة ومختلفة فقًا المضافة فيقدم الكسر الذي مقامه أعظم ويكون الأقل عن بينه منتخضا هه قليلا مثل خسة أكمان وثلاثة أكمان تمن وثلث تمن رئمها<sup>مد</sup> هكذا للكسر الذي مقامه أعظم ويكون الأقل عن بينه منتخضا هه قليلا مثل خسة أكمان وثلاثة أكمان تمن وثلث تمن رئمها<sup>مد</sup> هكذا

المختلفة فبصنها تحت بعض مثل خمسة أكمان وسنة أسباع رنمها هكذا

اً مون ت الففرة بإ السلا الاباب: لما الشيع الأبط الأعناداغضو اللانا الشارك أبوالتباس أخذ بو حمد بو مقماة الأرني رها الله وردني منا 1 بوت عمل الفاع" ويرح عمل القطاع" عما ورد بإ السلاة الابا

- ومدعما المع ورح عما الم
  - ا ۾ السخا الايا جا ۽ جد،
- " ﴿ السَّدَا قَابَةَ فَا : وَالْمُنَادَةَ وَالْمَنَادَ عَلَى مِيدَنَا عَمَدَ بِي وَمِيدَ، وَعَلَى أَنَا وَتُعرف كَقِرا
  - <sup>6</sup> () السخا قاباً رباط: مليا
- هُ وربد عَمَّةَ "ارُوسا" وبرح عَمَةَ "ارُوُسَلَ" عَمَا وردَقَ السَّحَةَ قَابَةَ وَقُو مَرْحُو"ا رُسَلًا"
  - " ﴿ السَّحَا فَايَاتِ : الْمُبْح
- » والسخة فابة ضاء: : وأسَّابة في ألاف الآلاف السَّاة وبرح شابًا كوبو، ما حور أشد والدور
  - <sup>9</sup> وربد عدة "خطرة" ويرج عدة "جنطة" عنا ورمع السخة الثابة

Figure 3: rumi numeral system in [1] page 1 printed

In the name of The God, the most Merciful, the most Compassionate The God prayed on Mohammed reigned and on his family and accompanied him and peace of delivery

The shortening from the work in the Rwmy in the calculation, formation of the participant virtuous of the magnificent jurist generous associated complete Abou AlEbas Ahmed Ben Mohammed Ben Othman al'azdy. He was introduced as mason's son numerical Almraakshy. The God sanctified went him and his uterus in his blessing and his casual dress and lengthens him.

The God praised neither his break-off for acne nor his end for border and the prayer on Mohammed is discerning and worshipped him and on his family and his peace and honor a lot. After, so this shortening from the work in the Rwmy in the account on towards what choice of the heads from the laborers fell, and from The God asks the good success.

Chapter in names ranks of the numbers and prescription glyphs in Rwmy.

Knows that the rank first is the ones nine and the second is the tens nine and the third is the hundreds nine and the fourth is the thousands nine and the fifth is the tens of thousands nine and the sixth is the hundreds of thousands nine and the seventh is the thousands thousands nine and so on until the infinite one.

For each number of the first three numbers ranks a sign witch distinguishes it, the first is one

and the last is nine hundred. The draws of thousands and its tens and its hundreds are the same as the draws of ones and tens and hundreds and the difference between them is the repetition. The form of the repetition is a bar under the number and these images for all them:

Likewise thousands of thousands and its tens and its hundreds return to glyphs before them and the difference between them the repetition. For each kind what be necessary for him likewise so on until the infinite one.

When the fractions are to be drawn, we write the base number which derives the fraction and draws above him a line called chair and writes above him the parts which

derives from him, and as the fraction's fraction. Here are examples of draw: one half  $\checkmark$ , two thirds  $\checkmark$ , tree quarters  $\checkmark$ , four fifths  $\checkmark$ , five sixths  $\checkmark$ , six sevenths  $\checkmark$ , seven eighths  $\checkmark$ , eight ninths  $\checkmark$ , nine tenths  $\checkmark$ . And leads the man working in make up provided that draw the one half in this way  $\checkmark$ , and one quarter in this way  $\checkmark$ and one third in this way  $\checkmark$ , and the two third in this way  $\checkmark$ . They don't use fractions

which there based are more than ten. If they arrive to have some of them they transform them to what you will know in the addition chapter after this one.

The fractions they used stubborn two kinds added and different. So for added fraction, the fraction whose based is great advances and the little about right and little low from him,

example five eighth and three eighth and third eighth in this way **FFF**. As for different fraction, parts are under others parts example five eighths and six sevenths in this way

Figure 4: rumi numeral system in [1] page 1 translated in english



ey.

Figure 5: Integer rumi symbols in [1]



Figure 6: Fraction rumi symbols in [1]



Figure 7: Examples in [3]

الجؤير هوانهايته مااتة < 1 R < e ωð La ωý 47 کی w, L, ۲ų 4 21 N でへ Np キシ ÷, s 1 544 25 SV S 6 ፟፟ቚ م ہ لر. • ^ Ē م •2 t., ٩/ 6 tor Ê7 51-21 21 6~ RA **ଜ**୍ୟ ď 25

Figure 8: Examples from [4]

parum dirutus, necesse fuit abscidere folia quæ inutilia evaserant; hæc suspitio confirmatur ex facto, quod folia octogesimo posteriora morsus vermium majores habent præcipue circa folium centesimum tertium; et notandum est hos morsus jam in codice existentes esse cum collatio, saltem secunda fiebat, nam quidam restaurati fuere, ut videtur ab antiquo et verba a vermibus plene corrosa in margine explicantur: charta antiquioris codicis major erat, nam a folio trigesimo nono usque ad centesimum quintum vestigia antiquioris foliorum numerationis existunt, nam numeratio ex magna parte abscisa fuit: in foliis secundæ partis nihil hujuscemodi numerationis distinguitur, et breves notæ marginales, præcipue verbum <u>uit</u> pluries in margine primæ partis scriptum fere evanuit.

x

SPECIMEN NUMERATIONIS FOLIORUN IN FODICE ESCURIALENSI.

Folierum numeratio. Codex habet foliorum numerationem modernam, factam postquam codex a librario numerationis arabicæ imperito compactus est; a folio enim secundo transilire necesse fuit ad folium decimum septimum et a folio vigesimo secundo iterum ad tertium recedere.

Ex foliorum numeratione antiqua codex centum sexaginta

Figure 9: Examples from [4]



Figure 10: Examples in [15] pages 50-51 from [4]



Figure 11: Examples in [15] pages 52-53 from [4]