# Rumi Numeral System Symbols, <br> Additional characters proposed to Unicode 

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## 1 Introduction

A special numeral system rumi ${ }^{1}$ has been in use in North Africa since the $\mathrm{X}^{\mathrm{e}}$ century. It remained in use until the XVIIe century. This system has been especially used in the administration of the city of Fez in Morocco. It has also been used in Al-Andalusians, Spain, starting from the XIIe century. The forms of the digits are quiet different from the Arabic ${ }^{2}$ or the Arabic-Indic ${ }^{2}$ digits in use today. The system of numeration was decimal, but not really positional. rumi use some special symbols (see Table 1, Table 2, Table 3 and Table 4). Some examples are available (see Table 5, Table 6 and Table 7). This system is also known as "zimam letters, Roman, Fez letters" (namely Hrwf al-zmAm, al-rumi, Hrwf fAs or also rasm al-zmam, Qalam al-rumi, Qalam al-fAsy).

## 2 Description

rumi numeral system has been described by many researchers and there is many studies about it. A deailed bibliography is presented in Figure 1 (see some figures from this bibliography in the end). We have adopted the one

[^0]described by the mathematician Ibn Al-Banna (1256-1321, Marrakech) in his famous book [1] "Abstract of using rumi in calculus" in Arabic (namely Al-YqtDAb mn al-Eml b-al-rwmI fI al-HsAb).
rumi use some special symbols for digits:

- rumi ones are(see Table 1): $\boldsymbol{J} \boldsymbol{S}$.


Multiples of thousand are represented by adding a slash under the based number:
- rumi thousands are noted by one bar under the number (see Table 5) (ex., $\boldsymbol{\sim}$ for three thousands);
- rumi million are noted by two bars under the number (see Table 6) (ex., $\boldsymbol{\sim}$ for three million);
- and so on.

Fraction is represented by adding a slash symbol separating the numerator from the denominator:

- rumi fractions are generally noted by (see Table 7):




## 3 proposition

These symbols are proposed to be included in Unicode Standard. There names are descriped in rumi ones digits (see Table 8), rumi tens digits (see Table 9) rumi handreds digits (see Table 10) and rumi specila fractions (see Table 11).

As many manuscripts in studying use these symbols, we need to encode them. We are working on studying and translating to english the Ibn AlBanna manuscript Al-YqtDAb mn al-Eml b-al-rwmI fI al-HsAb. A computer
system for transforming numbers from and to rumi numeral system is also in development.

The rumi and AntiSym fonts available, includes all these characters. In rumi font, used here, the shapes of the reference glyphs are scanned from [1]. It's in OpenType format [3] and converted in METAFONT as a ${ }^{A} T_{E X}$ package [4]. In AntiSym font, glyphs are drawing by hand in METAFONT as a LATEX package [2].

The shapes of the reference glyphs used are not frozen. They are continually being improved in Multilingual scientific e-document processing Project at Al-khawarizmi Atelier.

Some boxes are add to some symbols in Figures in order to emphases them and understand the purpose of the samples.

More information about this presentation is available in [5].

## References

[1] Ibn Al-Banna (1256-1321, Marrakech), Abstract of using rumi in calculus, in Arabic.
[2] Arabic mathematical old symbols package antisym for $\mathrm{LT}_{\mathrm{EX}}$, http://www.ucam.ac.ma/fssm/rydarab/system/zip/antisym.zip.
[3] rumi numeral system font in OpenType, http://www.ucam.ac.ma/fssm/rydarab/doc/unicode/rumi.ttf.
[4] rumi numeral system as a package for ${ }^{\mathrm{A}} \mathrm{T}_{\mathrm{E}} \mathrm{X}$, http://www.ucam.ac.ma/fssm/rydarab/system/zip/rumi.zip.
[5] Azzeddine Lazrek, Arabic mathematical symbols for Unicode, http://www.ucam.ac.ma/fssm/rydarab/english/unicode.htm.


Table 1: rumi ones symbols

| $10$ | $20$ | $\begin{array}{r} 30 \\ \end{array}$ | $\begin{aligned} & 40 \\ & 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & 50 \\ & \boldsymbol{Y} \end{aligned}$ | 60 | $70$ | 80 90 <br> c  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |

Table 2: rumi tens symbols

| 100 | 200 | 300 | 400 | 500 | 600 | 700 | 800 | 900 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $\boldsymbol{\infty}$ | $\boldsymbol{8}$ | 6 | 2 | 8 | 8 | 4 | $y$ | $\boldsymbol{2}$ |

Table 3: rumi handreds symbols

| 1/2 | 1/4 | $1 / 3$ | $2 / 3$ |
| :---: | :---: | :---: | :---: |
| jo |  |  | 4 |

Table 4: rumi special fractions symbols

| $1000$ | $\begin{array}{r} 2000 \\ 5 \end{array}$ | $\begin{aligned} & 3000 \\ & \boldsymbol{N} \\ & \hline \end{aligned}$ | $\begin{array}{r} 4000 \\ \hline \end{array}$ |  | $\begin{gathered} 6000 \\ \hline \end{gathered}$ | $\begin{gathered} 7000 \\ ? ~ \end{gathered}$ | $\begin{gathered} 8000 \\ \boldsymbol{b} \\ \hline \end{gathered}$ | $\begin{aligned} & 9000 \\ & 2 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 10000 \\ < \end{gathered}$ | $\begin{gathered} 20000 \\ \boldsymbol{w} \end{gathered}$ | $30000$ | N | $\begin{array}{r} 50000 \\ \boldsymbol{Y} \end{array}$ | $\begin{gathered} 60000 \\ \boldsymbol{z} \end{gathered}$ | $70000$ | $\begin{gathered} 80000 \\ 0 \end{gathered}$ | $\begin{gathered} 90000 \\ \frac{1}{2} \end{gathered}$ |
| $\begin{gathered} 100000 \\ \boldsymbol{e} \\ \hline \end{gathered}$ | $\begin{gathered} 200000 \\ \boldsymbol{\xi} \end{gathered}$ | $\begin{gathered} 300000 \\ \mathbf{C} \\ \hline \end{gathered}$ | $\begin{gathered} 400000 \\ \boldsymbol{Z} \\ \hline \end{gathered}$ | $500000$ | $\begin{gathered} 600000 \\ ? \end{gathered}$ | $\begin{gathered} 700000 \\ \text { 上 } \end{gathered}$ | $\begin{gathered} 800000 \\ \mathbf{y} \\ \hline \end{gathered}$ | $\begin{gathered} 900000 \\ \mathbf{2} \\ \hline \end{gathered}$ |

Table 5: rumi thousands examples

| $1000000$ | $2000000$ <br> 5 | 3000000 <br> ~N | $\begin{gathered} 4000000 \\ \boldsymbol{N} \end{gathered}$ | $$ | $6000000$ | $\begin{gathered} 7000000 \\ \boldsymbol{?} \end{gathered}$ | 8000000 $\zeta$ | 9000000 <br> 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $10000000$ | $20000000$ | $\begin{gathered} 30000000 \\ \downarrow \end{gathered}$ | $\begin{gathered} 40000000 \\ \infty \end{gathered}$ | $\begin{gathered} 50000000 \\ \boldsymbol{J} \end{gathered}$ | $60000000$ | $70000000$ | $80000000$ | 80000000 |
| 100000000 $?$ | $\begin{gathered} 200000000 \\ \boldsymbol{5} \end{gathered}$ | $\begin{gathered} 30000000 \\ \lll< \end{gathered}$ | $\begin{gathered} 400000000 \\ \vdots \\ \hline \hline \end{gathered}$ | $500000000$ | $600000000$ | $700000000$ | $800000000$ <br> $\geq$ | $900000000$ |

Table 6: rumi millions examples


Table 7: rumi fractions examples

」 RUMI DIGIT ONE
$\approx 00311$ digit one
$\approx 0661$ Arabic-Indic digit one
5 RUMI DIGIT TWO
$\approx 00322$ digit two
$\approx 0662$ r Arabic-Indic digit two
RUMI DIGIT THREE
$\approx 00333$ digit three
$\approx 0663 \upharpoonright$ Arabic-Indic digit three
入 RUMI DIGIT four
$\approx 00344$ digit four
$\approx 0664$ \& Arabic-Indic digit four
4 RUMI DIGIT FIVE
$\approx 00355$ digit five
$\approx 06650$ Arabic-Indic digit five
R RUMI DIGIT SIX
$\approx 00366$ digit six
$\approx 06667$ Arabic-Indic digit six
7 RUMI DIGIT SEVEN
$\approx 00377$ digit seven
$\approx 0667 \vee$ Arabic-Indic digit seven
$\boldsymbol{j}$ RUMI DIGIT EIGHT
$\approx 00388$ digit eight
$\approx 0668 \wedge$ Arabic-Indic digit eight
2 RUMI DIGIT NINE
$\approx 00399$ digit nine
$\approx 0669$ 9 Arabic-Indic digit nine
Table 8: RUMI ones digits

## < RUMI DIGIT TEN

- used as a symbol with a numeric value of 10 $\boldsymbol{\omega}$ RUMI DIGIT TWENTY
- used as a symbol with a numeric value of 20
$\downarrow$ RUMI DIGIT THIRTY
- used as a symbol with a numeric value of 30


## RUMI DIGIT FORTY

- used as a symbol with a numeric value of 40

RUMI DIGIT FIFTY

- used as a symbol with a numeric value of 50


## $₹$ RUMI DIGIT SIXTY

- used as a symbol with a numeric value of 60
- RUMI DIGIT SEVENTY
- used as a symbol with a numeric value of 70

〕. RUMI DIGIT EIGHTY

- used as a symbol with a numeric value of 80

RUMI DIGIT NINETY

- used as a symbol with a numeric value of 90
e RUMI DIGIT HUNDRED
- used as a symbol with a numeric value of 100

Table 9: RUMI tens digits

## 3 RUMI DIGIT TWO HUNDRED

- used as a symbol with a numeric value of 200
< RUMI DIGIT TREE HUNDRED
- used as a symbol with a numeric value of 300


## \& RUMI DIGIT FOUR HUNDRED

- used as a symbol with a numeric value of 400
§ RUMI DIGIT FIVE HUNDRED
- used as a symbol with a numeric value of 500
\& RUMI DIGIT SIX HUNDRED
- used as a symbol with a numeric value of 600

4 RUMI DIGIT SEVEN HUNDRED

- used as a symbol with a numeric value of 700
y RUMI DIGIT EIGHT HUNDRED
- used as a symbol with a numeric value of 800
z RUMI DIGIT NINE HUNDRED
- used as a symbol with a numeric value of 900

Table 10: RUMI handreds digits

RUMI FRACTION ONE HALF

- used as an other symbol with a numeric value of $1 / 2$
$\approx$ 00BD $1 / 2$ vulgar fraction one half
RUMI FRACTION ONE QUARTER
- used as a symbol with a numeric value of $1 / 4$
$\approx$ OOBC $1 / 4$ vulgar fraction one quarter
RUMI FRACTION ONE THIRD
- used as a symbol with a numeric value of $1 / 3$
$\approx 21531 / 3$ vulgar fraction one third
RUMI FRACTION TWO THIRDS
- used as an other symbol with a numeric value of $2 / 3$
$\approx 21542 / 3$ vulgar fraction two thirds
Table 11: RUMI special fractions




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Figure 1: rumi numeral system in [1] page 1


Figure 2: rumi numeral system in [1] page 1
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Figure 3：rumi numeral system in［1］page 1 printed

In the name of The God, the mot Merciful the nost Conqassionite The God prayed on Mohammed reigned and on his family and acoontamied him and peace of delivery

The shortening from the wook in the Romy in the calculation, fomation of the participint virtuous of the magnifioent jurist gererous associated complete Abou Allbbas Ahmed Ben Moharmed Ben Othman al'azdy. He was introduced as mason's son rumerical Almaakshy. The God sanctified went him and his utens in his blessing and his caval dress and lengthens him.
The God prased neither his break-off for acne nor his end for border and the prayer on Moharmed is disceming and worshipped him and on his family and his pease and howr a lot. After, so this shortening from the work in the Rwmy in the acovurt on towards what chore of the heads from the laboress fell, and from The God asks the good success.
Chapter in names ranks of the rumbers and prescription glyphs in Rumy.
Knows that the rank first is the ones nime and the seoond is the tens nire and the third is the lundreds rine and the fourth is the thowsands nine and the fifth is the tens of thowsands nime and the sixth is the homidreds of thousands nine and the severth is the thowsands thowsands nine and so on until the infinite one.
For each rumber of the frst thee rumbers ranks a sign witch ditinguishes it, the first is one and the last is nine hamdred. The draws of thowsands and its tens and is hundreds ane the same as the draws of ones and tens and loundreds and the difference between them is the repetiion. The form of the repetion is a bay under the number and these imazes for allthem:
Likenise thousands of thowsands and its tens and its loundreds retum to glyphs before them and the difference between them the repectition. For each kind what be necessayy for him likevise so onurtil the infnite ore.
When the faactions ane to be dramm, we write the base number which denives the fraction and draws above him a line called chair and wnites above him the parts which

derives from him, and as the faction's fration. Here are evanqles of draw: one halt $5^{5}$, two thinds $\omega^{2}$, tree quarters $\rho \infty$, four fifths $>^{-2}$, five sixths $r^{2}$, six severths $>^{\circ}$, seven eighths $57^{7}$, eight nimith $2^{2}$, nime terth $5^{24}$. And leads the man working in make up provided that draw the ore half in this way ${ }^{3} 5$, and one quarter in this way/s and one third in this way $/ \mathrm{w}$ and the two third in this way 40 . They don't vee fractions which there based are more than ten. If they amive to have some of them they tansform them to what you will low in the addition chapter after this one.
The factions they used stubbom two kinds added and different. So for added fraction, the fraction whose based is great adranoes and the little about night and litle low fom him,
example five eiglth and three eighth and thind eighth in this way fraction, parts are under others parts example fire eighths and six severths in this way


Figure 4: rumi numeral system in [1] page 1 translated in english


Figure 5: Integer rumi symbols in [1]


Figure 6: Fraction rumi symbols in [1]


Figure 7: Examples in [3]



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Figure 8: Examples from [4]
parum dirutus, necesse fuit abscidere folis quer inutilia evaserant, haec suspitio confirmatur ex facto, quod folia octogesimo posteriora morsus vermiam majores habent presipue circa folium centesimum tertium; et notandum est hos morsus jam in codice existentes esse cum collatio, saltem secunda fiebat, nam quidam restaurati fuere, ut videtur as antiquo et verba a vermibus plene corrosz in margine explicantur: charta antiquioris codicis major erat, nam a folio trigesimo nono usque ad centesimum quintum vestigia antiquioris foliorum numerationis existunt, nam numeratio ex magna parte abscisa fuit: in foliis secunde partis nihil hujuscemodi numerationis distinguitur, et breves notz marginales, precipue verbum pluries in margine prime partis scriptum fere evanuit.

Specimen numerationis foliorun in foobce Escurialensi.

$$
\begin{aligned}
& \left.2_{1}^{3}\{2\}_{k}^{v} \text { 2\}, }\{e\}_{5}\right\}_{1}
\end{aligned}
$$

Folioruw numeratio. Codex habet folioram numerationem modernam, factam postquam codex a librario numerationis arabica imperito compactus est; a folio enim secundo transilire necesse fuit ad folium decimum septimum e: a folio vigesimo secundo iterum ad tertium recedere.

Ex foliorum numeratione antiqua codex centum sexaginta
Figure 9: Examples from [4]


Figure 10: Examples in [15] pages 50-51 from [4]
ARV, Clero, libro 3850.

Figure 11: Examples in [15] pages 52-53 from [4]


[^0]:    ${ }^{1}$ using Transtec Transliteration
    http://www.ucam.ac.ma/fssm/rydarab/doc/communic/transtec.pdf
    ${ }^{2}$ the identifier name used by The Unicode Consortium http://www.unicode.org

